Name ______________________________ Date ________ Period ________

HARDY-WEINBERG EQUILIBRIUM

Equilibrium in a population occurs when there is NO LONGER any evolution taking place in a population. There are 5 requirements for equilibrium. They are as follows:

1. No mutations can take place. Mutations are changes in the DNA which can cause the trait to become favorable or non-favorable.

2. Mating must be random. Random mating assures that the trait is neither favorable or non-favorable.

3. Natural selection cannot take place. This assures that the trait is not going to become favorable or non-favorable.

4. There is no migration into or out of the population. This makes sure that the gene pool is not changed, resulting in different % of genes in the population.

5. The population must be large. Theoretically, it should be "unlimited" in size. This assures that random events (such as a hurricane or tornado) do not affect the gene pool.

FORMULA ONE: $p + q = 1$
FORMULA TWO: $p^2 + 2pq + q^2 = 1$

p = the dominant allele, usually indicated by a capital letter (such as "B")
q = the recessive allele, usually indicated by a lowercase letter (such as "b")
p^2 = the population of homozygous dominant individuals (those with "BB")
$2pq$ = the population of heterozygous dominant individuals (those with "Bb")
q^2 = the population of homozygous recessive individuals (those with "bb")

Exercise 6 (Modules 13.7–13.8)

These modules introduce evolution of populations. After reading the modules, circle the word that best matches each statement

1. A group of individuals of the same species: *gene pool, population*
2. What natural selection acts on: *individual, population*
3. What actually evolves: *individual, population*
4. All the alleles in all the individuals in the population: *genome, gene pool*
5. Change in relative frequencies of alleles in the gene pool: *microevolution, macroevolution*
6. Causes of variation (circle all that apply): *mutation, natural selection, sexual reproduction*
7. Portion of variation relevant to natural selection: *acquired, genetic*
8. Produces new alleles: *mutation, sexual reproduction*
9. Where most mutations occur: *body (somatic) cells, gametes*
10. Mutations that affect population's variability: *somatic mutations, mutations in gametes*
11. Most mutations: *harmful, helpful*
12. Generates most new gene combinations in animals and plants: *mutation, sexual reproduction*

Exercise 7 (Modules 13.9–13.10)

Microevolution is the change of frequencies of alleles in the gene pool. To see what happens when microevolution occurs, it is helpful to first look at a hypothetical population that is *not* evolving—a population at "Hardy-Weinberg equilibrium." Imagine a population of 100 annual wildflowers, some red and some yellow. The red allele, *R*, is dominant; the yellow allele, *r*, is recessive. There are 36 *RR* plants in the population, 48 *Rr* plants, and 16 *rr* (yellow) plants. If the population is at Hardy-Weinberg equilibrium, what will be the frequencies of the various genotypes and the frequencies of the two alleles, *R* and *r*, in the next generation? Follow the example in Module 13.9 as a guide, and fill in the blanks below to figure out the frequencies for this example.

First, figure out genotype frequencies for the current generation:

A. Phenotypes	Red	Red	Yellow
B. Genotypes	____	____	____
C. Number of plants (total 100)	____	____	____
D. Genotype frequencies (number of genotypes/100)	____	____	____

Next, figure out the frequencies of *R* and *r* alleles in the gene pool:

E. Number of *R* alleles in gene pool	____	____	____
F. Number of *r* alleles in gene pool	____	____	____

G. Allele frequencies (number of *R* alleles/200 or number of *r* alleles/200) Frequency of *R*: p = ____ Frequency of *r*: q = ____

Now you know the frequency of *R* and *r* gametes these plants will produce:

H. Gamete frequencies (= allele frequencies) Frequency of *R*: ____ Frequency of *r*: ____

Now you can use the rule of multiplication to calculate the frequencies of the three possible genotypes of plants in the second generation:

I. Phenotype	Red	Red	Yellow
J. Genotype	____	____	____
K. Genotype frequencies	____	____	____

Now you can figure out the frequencies of *R* and *r* alleles in the gene pool for the second generation (assuming the population stays at 100 individuals):

L. Number of *R* alleles in gene pool	____	____	____
M. Number of *r* alleles in gene pool	____	____	____

N. Allele frequencies (number of *R* alleles/200 or number of *r* alleles/200) Frequency of *R*: p = ____ Frequency of *r*: q = ____

O. What happened to the genotype and allele frequencies in the second generation?

What would you predict for the third generation? Why?

P. Did the number of red plants or red alleles increase? Isn't the red allele dominant? Explain.

ALBINISM: A SAMPLE HARDY-WEINBERG PROBLEM

Albinism is a rare genetically inherited trait that is only expressed in the phenotype of homozygous recessive individuals (aa). The most characteristic symptom is a marked deficiency in the skin and hair pigment melanin. This condition can occur among any human group as well as among other animal species. The average human frequency of albinism in North America is only about 1 in 20,000.

Referring back to the Hardy-Weinberg equation ($p^2 + 2pq + q^2 = 1$), the frequency of homozygous recessive individuals (aa) in a population is q^2. Therefore, in North America the following must be true for albinism:

$$q^2 = 1/20{,}000 = .00005$$

By taking the square root of both sides of this equation, we get: (Note: the numbers in this example are rounded off for simplification.)

$$q = .007$$

In other words, the frequency of the recessive albinism allele (a) is .00707 or about 1 in 140. Knowing one of the two variables (q) in the Hardy-Weinberg equation, it is easy to solve for the other (p).

$$p = 1 - q$$
$$p = 1 - .007$$
$$p = .993$$

The frequency of the dominant, normal allele (A) is, therefore, .99293 or about 99 in 100.

The next step is to plug the frequencies of p and q into the Hardy-Weinberg equation:

$$p^2 + 2pq + q^2 = 1$$
$$(.993)^2 + 2\,(.993)(.007) + (.007)^2 = 1$$
$$.986 + .014 + .00005 = 1$$

This gives us the frequencies for each of the three genotypes for this trait in the population:

p^2 =	predicted frequency of homozygous dominant individuals	= .986 = 98.6%
$2pq$ =	predicted frequency of heterozygous individuals	= .014 = 1.4%
q^2 =	predicted frequency of homozygous recessive individuals (the albinos)	= .00005 = .005%

With a frequency of .005% (about 1 in 20,000), albinos are extremely rare. However, heterozygous carriers for this trait, with a predicted frequency of 1.4% (about 1 in 72), are far more common than most people imagine. There are roughly 278 times more carriers than albinos. Clearly, though, the vast majority of humans (98.6%) probably are homozygous dominant and do not have the albinism allele.

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POPULATION GENETICS: HOW TO SOLVE HARDY WEINBERG PROBLEMS

Hardy and Weinberg stated that Nature really wants things to be balanced (meaning not changing). However, we know that Nature is constantly changing. So, when one thing in nature changes, it usually leads to a lot of other things having to change.

Hardy and Weinberg came up with a hypothesis/idea that explained how populations could grow or change. It had 5 parts to it.

In order for a population to be at equilibrium (not changing), it must

1. Have no mutations occur
2. Have nothing moving in (immigration) or moving out (emigration)
3. Have no natural selection
4. Have a large breeding population
5. Random mating

Now, you know that nature changes. Lets look at how/why.

1. Mutations (changes in DNA) happen every second of every day. We have no way of controlling mutations in nature. Anytime some mutates, it causes a change. And if a change occurs, then the population is not balanced.

2. Things constantly move in and out of areas. It would be like you having to stay in your classroom 24 hours a day for the rest of your life. Bacteria move in and out of a room in the air. So, you know it is very hard to stop things from moving (unless you are on an island far, far, away.

3. Natural selection is what you may call "survival of the fittest." Depending on the circumstances, there will be competition for food, shelter, mates, etc. And the one best suited will live to reproduce, while the other will die or leave. Thus, losing an entire set of genetic information that could play a role in how that population survives.

4. Nature likes to have lots of options; thus, it would like to have a large breeding population. Meaning, a lot of people, for example, to chose from. However, you know that things tend to mate within their area. A group of bird in one area of the forest is going to stay in that area of the forest. Thus, leading to a small breeding population.

5. Everything has a certain "type" that he/she/it is looking for. Meaning, you might find people with blond hair more attractive than people with brown hair. So, you would only see people with blond hair. That is not random. You are picking a trait you like. So, random mating is uncommon.

SOLVING POPULATION PROBLEMS

Hardy-Weinberg states:

P= the frequency of the dominant allele (so the % of A are in the population)
Q= the frequency of the recessive allele (so the % of a in a population)

So p + q = 1

Using the equation:
$p^2 + 2pq + q^2 = 1$

p^2 = frequency of *AA* (homozygous dominant)
$2pq$ = frequency of *Aa* (heterozygous)
q^2 = frequency of *aa* (homozygous recessive)

So… SOLVE THESE PROBLEMS!!! ☺

1. In a certain population of 1000 fruit flies, 640 have red eyes while the remainder have sepia eyes. The sepia eye trait is recessive to red eyes. How many individuals would you expect to be homozygous for red eye color?

2. If you observe a population and find that 16% show the recessive trait, you know the frequency of the *aa* genotype. This means you know q^2. What is q for this population?

3. If 9% of an African population is born with a severe form of sickle-cell anemia (*ss*), what percentage of the population will be more resistant to malaria because they are heterozygous (*Ss*) for the sickle-cell gene?

4. There are 100 students in a class. Ninety-six did well in the course whereas four blew it totally and received a grade of F. Sorry! In the highly unlikely event that these traits are genetic rather than environmental, AND if the traits represent dominant and recessive alleles, AND if the 4% represent the frequency of the homozygous recessive condition (q^2), please calculate the following…

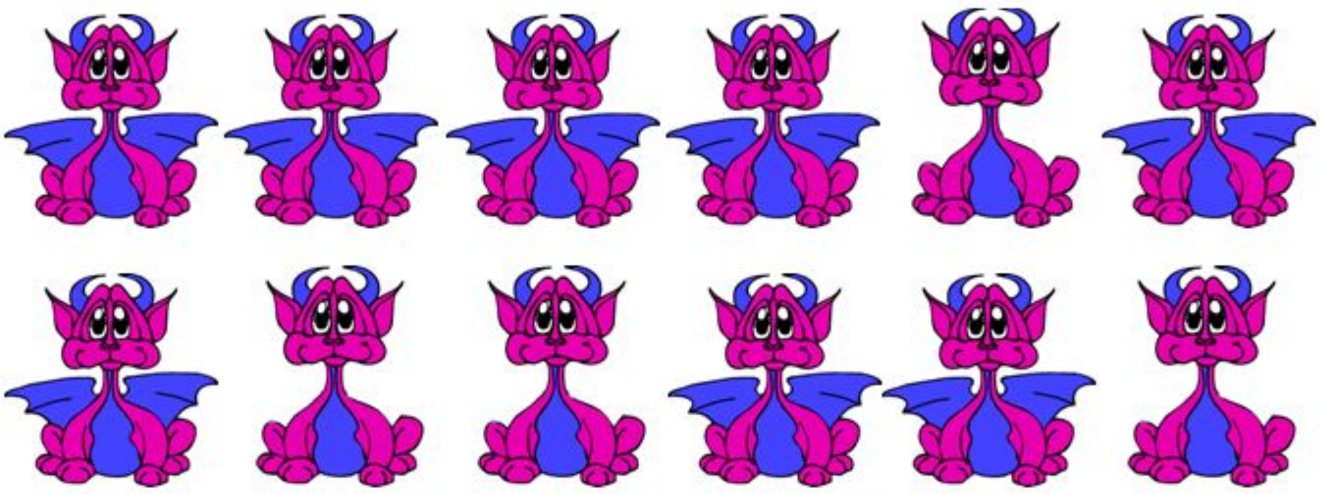
    a. the frequency of the recessive allele (q)
    b. the frequency of the dominant allele (p)
    c. the frequency of the heterozygous dominant individuals ($2pq$)
    d. the frequency of the homozygous dominant individuals (p^2)

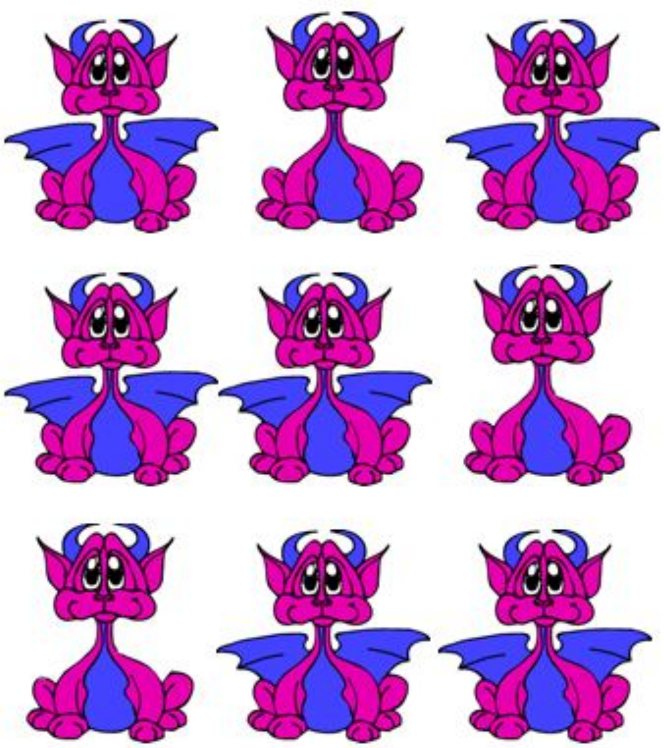
Name_______________________________________

Hardy Weinberg Problem Set

p2 + 2pq + q2 = 1 and p + q = 1
p = frequency of the dominant allele in the population
q = frequency of the recessive allele in the population
p2 = percentage of homozygous dominant individuals
q2 = percentage of homozygous recessive individuals
2pq = percentage of heterozygous individuals

PROBLEM #1. - View the Dragons below. The winged trait is dominant





q^2 (aa) = ____________

q (a) = _____________

p (A) = _____________

p^2 (AA) = ____________

2pq (Aa) = ____________

What is the frequency of heterozygote dragons in your population?

____

PROBLEM #2

You have sampled a population in which you know that the percentage of the homozygous recessive genotype (aa) is 36%. Using that 36%, calculate the following:
A. The frequency of the "aa" genotype.
B. The frequency of the "a" allele.
C. The frequency of the "A" allele.
D. The frequencies of the genotypes "AA" and "Aa."
E. The frequencies of the two possible phenotypes if "A" is completely dominant over "a."

PROBLEM #3

There are 100 students in a class. Ninety-six did well in the course whereas four blew it totally and received a grade of F. Sorry. In the highly unlikely event that these traits are genetic rather than environmental, if these traits involve dominant and recessive alleles, and if the four (4%) represent the frequency of the homozygous recessive condition, please calculate the following:
A. The frequency of the recessive allele.
B. The frequency of the dominant allele.
C. The frequency of heterozygous individuals.

PROBLEM #4.

Within a population of butterflies, the color brown (B) is dominant over the color white (b). And, 40% of all butterflies are white. Given this simple information, which is something that is very likely to be on an exam, calculate the following:
A. The percentage of butterflies in the population that are heterozygous.
B. The frequency of homozygous dominant individuals.

PROBLEM #5

After graduation, you and 19 of your closest friends (lets say 10 males and 10 females) charter a plane to go on a round-the-world tour. Unfortunately, you all crash land (safely) on a deserted island. No one finds you and you start a new population totally isolated from the rest of the world. Two of your friends carry (i.e. are heterozygous for) the recessive cystic fibrosis allele (c).

Assuming that the frequency of this allele does not change as the population grows, what will be the incidence of cystic fibrosis on your island? ______

PROBLEM #6

Cystic fibrosis is a recessive condition that affects about 1 in 2,500 babies in the Caucasian population of the United States. Please calculate the following.
The frequency of the recessive allele in the population. ______
The frequency of the dominant allele in the population. ______
The percentage of heterozygous individuals (carriers) in the population. ____

Problem #7. REAL WORLD APPLICATION PROBLEM

Choose a human trait to study and survey a population at your school. (Aim for at least a sample size of 50 to get meaningful results). Use your sample to determine the allele frequencies in the human population.

Traits (dominant listed first)

Hitchhiker's Thumb vs Straight Thumbs
Widow's peak vs straight hairline
PTC taster vs non-taster
Short Big toe vs long big toe
Free earlobes vs attached earlobes
Tongue rolling vs non-rolling
Bent little fingers vs straight little fingers
Arm crossing (left over right) vs right over left
Ear points vs no ear points

Hardy-Weinberg problem set

Hardy-Weinberg Theorem states that if a population is NOT evolving then the frequencies of the alleles in the population will remain stable across generations - it is in equilibrium.

We can use the Hardy-Weinberg equation to make predictions about the relative frequency of the different alleles (as well as the associated genotypes), even if there is population growth, as long as the five conditions we discussed in class hold true. Think about what those five conditions are.........

Alternatively, Hardy-Weinberg equation can be a means to determine if a population is indeed evolving - that the allele frequencies are changing and therefore NOT at equilibrium.

The trick to using the Hardy-Weinberg equation to help evaluate the frequency of a particular allele frequency at time X, or to make a prediction about the frequency of a particular genotype/phenotype in future generations, is to go through the math is a step-by-step manner.

For the problems below assume all genes have only two alleles and there is a simple dominant recessive relationship.

1st question is the SAME QUESTION AS FOUND IN YOUR COURSE PACKET --

1. If 98 out of 200 individuals in a population express the recessive phenotype, what percent of the population would you predict would be heterozygotes?

(a) I have given you information on the frequency of the homozygous recessive (or q*2*). So start by determining q*2* and then solving for q.

(b) Now that you have q, you can solve for p. Remember there are only two alleles in the population, so if you add the frequency of the two alleles, you have accounted for all possibilities and it must equal 1. So p + q = 1.

(c) Now what is the formula for heterozygotes? Think back to the Hardy-Weinberg equation -- it is dealing with the genotypes of individuals in the population.

(d) Now that you have figured out the % of heterozygotes, can you figure out the % of homozygous dominant? Does the % of homozygous dominant, heterozygotes and homozygous recessive individuals add up to 100%? If not, you have made an error. Those are the only three genotypes possible with only two alleles and a simple dominant and recessive relationship.

NEW QUESTIONS --

2. Your original population of 200 was hit by a tidal wave and 100 organisms were wiped out, leaving 36 homozygous recessive out of the 100 survivors. If we assume that all individuals were equally likely to be wiped out, how did the tidal wave affect the predicted frequencies of the alleles in the population? NOTE: assume the new population is at equilibrium -- AFTER the event - so you are comparing two populations what are at equilibrium to look for changes in allele frequencies.

Again, start with the frequency you know -- homozygous recessive. Follow the same step-by-step procedure as above.

What is the frequency of homozygous recessive?

What is the predicted frequency of heterozygotes?

What is the predicted frequency of homozygous dominant?

Given that the allele frequencies did change as the result of the tidal wave, we would say that microevolution has occurred. What do we call the phenomenon that caused this evolution?

NOTE -- I am not necessarily asking the questions in the order in which you should answer them, given the step-by-step approach.

3. Lets say that brown fur coloring is dominant to gray fur coloring in mice. If you have 168 brown mice in a population of 200 mice........

What is the predicted frequency of heterozygotes?

What is the predicted frequency of homozygous dominant?

What is the predicted frequency of homozygous recessive?

----

4. ***If 81% of a population is homozygous recessive for a given trait..........***

What is the predicted frequency of homozygous dominant?

What is the predicted frequency of heterozygotes?

What is the frequency of the dominant and recessive alleles in the population?

----

5. ***If 51% of the population carries at least one copy of the recessive allele.......***

What is the predicted frequency of individuals in the population that express the dominant phenotype?

What is the predicted frequency of individuals in the population that express the recessive phenotype?

ANSWERS -- work through the problems first! Remember to always double-check yourself by adding the frequencies of the three genotypes.